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Elastic scattering of γ rays and x-rays – a progress report

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Abstract: Experimental and theoretical studies of elastic scattering of γ rays and high energy x-rays are reviewed in order to focus attention on the present understanding of Rayleigh scattering from bound electrons, of Delbrück scattering from vacuum fluctuations, and of nuclear scattering. A few desirable directions of further work are also indicated.

Keywords: Rayleigh and Delbrück scattering of gamma rays

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1. Introduction

Substantial progress has been achieved during the previous four decades in our understanding of elastic scattering of energetic photons by neutral atoms in their ground states. Elastic scattering by bound atomic electrons is usually called Rayleigh scattering. Photon scattering by the nucleus and by vacuum fluctuations in the strong Coulomb field of the nucleus also contribute to the elastic scattering amplitude. The last mentioned process, called Delbrück scattering and viewed classically as arising from the interaction of the electromagnetic field of incident radiation with the Coulomb field of the nuclear charge, is an important consequence of quantum electrodynamics. It has a deep connection with vacuum polarization and with photon-photon scattering phenomena.

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Sec. 2 presents a brief description of experimental techniques used in the measurements of cross sections for elastic scattering of photons of energies larger than about 25 keV. Special high-resolution techniques employed at lower energies are omitted from this report. The theoretical background is outlined in Sec. 3. In Sec. 4, the results of some experimental investigations are highlighted in order to clarify the present understanding of the different processes mentioned earlier. Some of the further studies needed in this field are indicated briefly in Sec. 5.

2. Experimental Techniques

In the case of photon energies less than about 4 MeV, naturally occurring radioactive isotopes (e.g. Th C′′ or ^{208}Tl), sources produced by neutron activation (e.g. ^{60}Co) or by fission reactions (e.g. ^{137}Cs) or by charged particle reactions (e.g. ^{24}Na, ^{56}Co, ^{66}Ga) have been used. Recently, synchrotron x-rays of energies less than about 50 keV have also been used. In-beam (n,γ) reactions caused by neutrons from nuclear reactors have been employed for producing photons of energies up to about 12 MeV. Bremsstrahlung from accelerated electrons is an important source of photons up to the highest energies investigated so far (~ 7 GeV). Photons arising from annihilation in flight of accelerated positrons, and "tagged" bremsstrahlung photons in coincidence with radiating electrons have been utilised from 15 MeV to about 800 MeV.

In the energy range below about 15 MeV, NaI(Tl) detectors were used till about 1970 but have been replaced since then by the higher resolution Ge(Li) or HpGe detectors. Since intrinsic detection efficiencies of commercially available germanium detectors above about 20 MeV is very small, less than 0.5%, it is necessary to use in such cases NaI(Tl) detectors of very large size in spite of their poorer resolution. In the GeV range, e^+-e^- pairs produced by scattered high energy photons were momentum analysed by magnetic deflection and detected by telescopes consisting of multi-wire-proportional-chambers (MWPC) and scintillation counters [1].

A multi-energy ^{133}Ba source was used in a recent study of elastic scattering of 81.0 keV γ rays [2]. The resulting scattered photon spectrum in the neighbourhood of 81 keV is shown as an illustration in Fig. 1. It was possible by standard procedures to determine the elastic scattering counts, N_{el}, which are related to the differential cross section of the target atom for elastic scattering through Eq. (1).

$$N_{el} = \frac{S}{4\pi}\frac{n_T}{V}\varepsilon_D \int_{\text{detector}} d\Omega_D \int_{\text{target}} dV \frac{1}{r_1^2} \exp\left[-\mu\left(x_1 + x_2\right)\right] \frac{d\sigma_{el}}{d\Omega}, \tag{1}$$

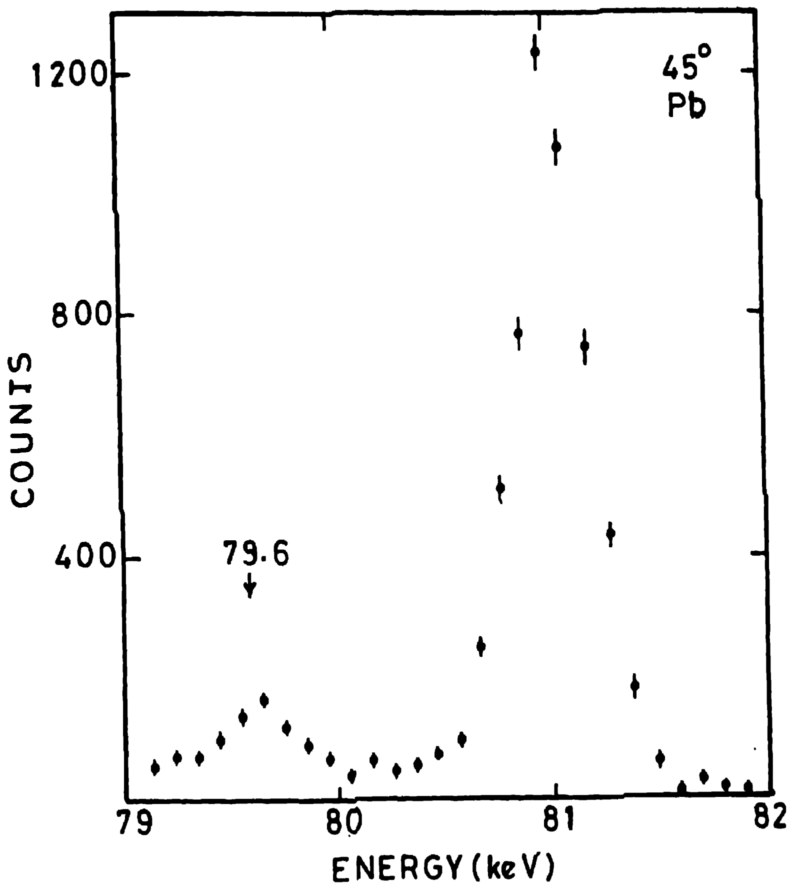

Fig. 1. HpGe detector spectrum near 81 keV of ^{133}Ba γ rays scattered by lead througl 45° [2]. The 81 keV elastic scattering peak is clearly separated from the weaker 79.6 keV peak

where S is the number of γ rays emitted per second by an isotropic source, n_T / V is the number of target atoms per unit volume, r_1 is the distance from the source to a target element of volume dV, μ is the attenuation coefficient of the target, x_1 and x_2 are the distances traversed in the target by the incident and the scattered γ rays, respectively, and ε_D is the efficiency of the detector. In order to ensure a small angular acceptance, the target dimensions are usually chosen to be small in comparison to r_1 and to the distance r_2 from the target to the detector. In such circumstances, Eq. (2) is a good approximation to Eq. (1).

$$N_{\mathrm{el}} \cong \frac{S}{4\pi}\, \varepsilon_{\mathrm{D}}\, n_{\mathrm{T}}\, (\Omega_{\mathrm{D}})_{\mathrm{av}} \left(\frac{T_{\mathrm{el}}}{r_1^2} \right)_{\mathrm{av}} \frac{d\sigma_{\mathrm{el}}}{d\Omega}, \tag{2}$$

where T_{el} and Ω_{D} indicate the target transmission and the solid angle subtended by the detector at the target, respectively, and the subscript *av* indicates an average over the target dimensions. Note that thin targets, of thickness less than about $0.2/\mu$, should be used in order to minimise possible effects of multiple scattering. The error in the value of $d\sigma_{\mathrm{el}}/d\Omega$ is minimsed by the avoidance of a direct measurement of the strong source intensity according to one of the following three methods.

In the first method, counts N_{W} are determined with a weak source of intensity W at the target position. Then, we get

$$\frac{N_{\mathrm{el}}}{N_{\mathrm{W}}} = \frac{S}{W}\, n_{\mathrm{T}} \left(\frac{T_{\mathrm{el}}}{r_1^2} \right)_{\mathrm{av}} \frac{d\sigma_{\mathrm{el}}}{d\Omega}. \tag{3}$$

Note that ε_{D} and Ω_{D} are eliminated by this procedure. In an auxiliary experiment, a determination is made of the intensity W relative to the intensity S after transmission through a sufficient thickness of an absorber of known attenuation coefficient. It is difficult to achieve an accuracy of better than $\pm 4\%$ in the value of the ratio S/W.

If an accelerator is used to generate photons, the accelerator beam current is reduced by a precisely known large factor f and the reduced beam intensity N_f is determined so that

$$S = f N_f. \tag{4}$$

During the measurement of N_f, the factor f has to be typically larger than 10^4 in order minimise counting losses due to dead time. It is difficult to reduce the error in f to less than a few % over such a large range of values.

In the third method, counts N_{el} are compared with Compton scattering counts obtained at the same angle with a low-Z target such as that of aluminium.

$$\frac{N_{\mathrm{el}}}{N_{\mathrm{Comp}}^{\mathrm{Al}}} = \frac{n_{\mathrm{T}}}{n^{\mathrm{Al}}} \left(\frac{T_{\mathrm{el}}}{T_{\mathrm{Comp}}^{\mathrm{Al}}} \right)_{\mathrm{av}} \left(\frac{\varepsilon_{\mathrm{D}}}{\varepsilon_{\mathrm{Comp}}} \right)_{\mathrm{av}} \left(\frac{d\sigma_{\mathrm{el}}}{d\Omega} \Big/ \frac{d\sigma_{\mathrm{Comp}}^{\mathrm{Al}}}{d\Omega} \right), \tag{5}$$

where n^{Al} is the number of aluminium atoms, the average transmission factor for Compton scattered photons in the aluminium target is $(T^{\mathrm{Al}}_{\mathrm{Comp}})_{\mathrm{av}}$, and $(\varepsilon_{\mathrm{Comp}})_{\mathrm{av}}$ is the average detection efficiency for Compton scattered photons.
Further,

$$\frac{d\sigma^{\mathrm{Al}}_{\mathrm{Comp}}}{d\Omega} = \frac{d\sigma^{\mathrm{KN}}}{d\Omega} S(x, Z=13), \tag{6}$$

where the first factor on the right hand side is the Klein-Nishina prediction for Compton scattering by a free electron, the second factor is the incoherent scattering function for aluminium, $x = \sin(\theta/2)/\lambda$ and λ is the wavelength of the incident radiation. Values of $S(x, Z)$ have been tabulated over an extensive range of x [3], and independently with a finer grid up to 4 Å^{-1} [4]. Note that, with this method, absolute values of transmission factors and detection efficiencies are not needed, and the effects of beam profile over a target are minimised. However, in the case of a multi-energy source such as ^{133}Ba, systematic errors in the determination of $N^{\mathrm{Al}}_{\mathrm{Comp}}$ are of the order of a few per cent.

3. Theoretical Background

A. General discussion.

The elastic scattering amplitude is usually partitioned into components corresponding to the three processes mentioned in the Introduction. In some discussions, the nuclear amplitude is further partitioned into a nuclear Thomson component, representing only the scattering from the nucleus treated as a point charge, and the so-called nuclear resonance component.

$$\begin{aligned} A_{\mathrm{el}} &= A_{\mathrm{Rayleigh}} + A_{\mathrm{nuclear}} + A_{\mathrm{Delbrück}} \\ &= A_{\mathrm{R}} + A_{\mathrm{N}} + A_{\mathrm{D}} \\ &= A_{\mathrm{R}} + (A_{\mathrm{NT}} + A_{\mathrm{NR}}) + A_{\mathrm{D}} \, . \end{aligned} \tag{7}$$

Each of the three amplitudes has real (r) and imaginary (i) parts. In the plane polarisation, and circular polarisation descriptions, the independent amplitudes are written as $(A_{\parallel}, A_{\perp})$ and (A_{++}, A_{+-}), respectively, where

$$A_{++} = \frac{A_{\parallel} + A_{\perp}}{2}, A_{+-} = \frac{A_{\parallel} - A_{\perp}}{2}. \tag{8}$$

Thus, there are in general twelve scattering amplitudes and the differential cross section for elastic scattering of unpolarised photons is given by Eq.(9).

$$\frac{d\sigma_{\mathrm{el}}}{d\Omega} = \frac{1}{2}\left(|A_{\parallel}|^2 + |A_{\perp}|^2\right) = \left(|A_{++}|^2 + |A_{+-}|^2\right). \tag{9}$$

The complexity of the calculations is reduced significantly by noting relations (10) to (15). An important limit of $\mathrm{Re}A_{\mathrm{N}}$ was deduced some time ago [5,6,7] on the basis of invariance with respect to Lorentz, gauge and charge-conjugation transformations.

$$\text{For } h\nu \ll m_{\pi}c^2,\ \mathrm{Re}\,A_{\mathrm{N}} \approx A_{\mathrm{NT}} \tag{10}$$

With the help of the well known optical theorem and dispersion relations [8], the amplitudes for Rayleigh, nuclear and Delbrück scattering are shown to be related to atomic photoabsorption, nuclear photoabsorption, and $(e^+\text{-}e^-)$ pair production, respectively. Since a chance overlap of nuclear γ energies with excitation energies of bound nuclear levels is extremely rare, in most cases we get relation (11).

$$\text{For } h\nu < (\gamma, n) \text{ threshold, } \mathrm{Im}\,A_{\mathrm{NR}} \sim 0 \tag{11}$$

Similarly, the connection of $\mathrm{Im}\,A_{\mathrm{D}}$ with pair production leads to Eq.(12).

$$\text{For } h\nu \le 1\ \mathrm{MeV},\ \mathrm{Im}\,A_{\mathrm{D}} = 0. \tag{12}$$

Further,

$$\text{for high } h\nu,\ \mathrm{Re}\,A_{\mathrm{D}} \text{ is small,} \tag{13}$$

and

$$\mathrm{Im}\,A_{\mathrm{D}} \text{ decreases with increasing } x. \tag{14}$$

In the form-factor (FF) or modified-relativistic-form-factor (MF) approximations, Re A_R decreases with increasing x and with decreasing Z. (15)

The relative phase relations between the different amplitudes have to be taken into account in the calculation of scattering cross sections. For example, at 0° there are constructive interference between A_{NT} and Re A_R, and destructive interferences between A_{NT} and Re A_{NR} for $h\nu < 10$ MeV, and between Re A_R and Re A_D. Similar phase relations hold in the case of the perpendicular amplitudes at other angles.

B. Rayleigh scattering

The non-relativistic $(e^2/2mc^2)A^2$ term in the interaction of a charged particle of charge e and mass m with the vector potential A of the electromagnetic radiation leads in the form-factor (FF) approximation to a dependence of the elastic scattering cross section on x. The form factor is effectively a Fourier transform of the spatial charge distribution. Sometimes relativistic wave functions are used for the electron states. The relativistic form factor (RFF) frequently results in a poorer approximation to the elastic scattering cross sections than the FF, since partially cancelling contributions from higher-multipoles and retardation effects are not properly taken into account merely by the use of relativistic wave functions. The relativistic modified form factor (MF) is introduced to take into account in an approximate way electron binding in intermediate states. The MF approximation is found to be satisfactory when $h\nu \gg B_K$ and x is small, where B_K is the K shell electron binding energy. The $(-e/mc)\boldsymbol{p} \bullet \boldsymbol{A}$ term in the interaction Hamiltonian leads to explicitly energy dependent or "anomalous" scattering factors (ASF) which have been usually assumed to be independent of angle [9,10,11].

The form factor of the lead K shell is shown as a function of x in Fig. 2. Note that in this case, the FF begins to drop at about 3 Å^{-1}, attains about 50% of its maximum value at about 15 Å^{-1}, and approaches about 0.02 in the neighbourhood of 80 Å^{-1}. The last mentioned regime is relevant for the discussion in Sec. 4 of large-angle scattering of 1.33-MeV γ rays. Let $K = \boldsymbol{k}_i - \boldsymbol{k}_f$, where $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ are the wave vectors of the incident and the scattered radiations, respectively, and $K = 4\pi x$. Thus when $x = 15$ Å^{-1}, $K = 187$ Å^{-1} and $KR \sim 1.15$, where R the radius ($\sim 6 \times 10^{-11}$ cm) of the lead K shell. Similarly, the form factor for the lead nucleus with a radius of about 6×10^{-13} cm will show a sharp drop when x is larger than about 1500 Å^{-1}. The form factors for the outer electron shells with increasingly diffuse charge distributions begin to drop at x values much smaller than that mentioned above for the K shell.

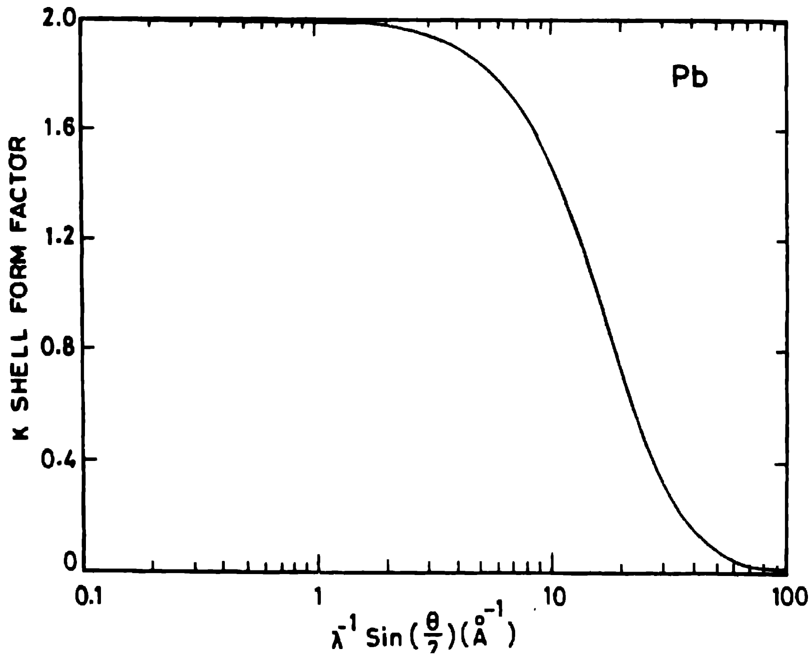

Fig. 2. The form factor (FF) of lead K shell as a function of x, where $x = \sin(\theta/2)/\lambda$. See Sec. 3 for comments.

A convenient unit for Rayleigh scattering amplitudes is $r_0 = e^2/mc^2$, i.e., about 2.8×10^{-13} cm. The corresponding unit r_N for the nuclear-Thomson scattering amplitude is given by Eq. (16).

$$r_N = \frac{Z^2 e^2}{AM_p c^2} = \frac{e^2}{M_p c^2}\left(Z - \frac{NZ}{A}\right), \tag{16}$$

where M_p is the proton mass and A is the atomic weight. So

$$r_N \sim 7.5 \times 10^{-17} Z \text{ cm for small } Z\text{, i.e.,} < 10^{-15} \text{ cm for } Z \leq 13,$$
$$r_N \sim 6 \times 10^{-17} Z \text{ cm for high } Z\text{, i.e.,} \sim 5 \times 10^{-15} \text{ cm for } Z \sim 82. \tag{17}$$

In the relativistic theory, the interaction term is $-e\boldsymbol{\alpha} \bullet \boldsymbol{A}$, where $\boldsymbol{\alpha}$ is the well known 4x4 Dirac velocity operator. The relativistic second order S-matrix calculations initiated by Brown et al. [12] and pursued by Johnson et al. [13] have been developed considerably by the Pittsburgh group [14]. The last two sets of calculations take account of screening in the initial, intermediate and final electron states. Since in many cases, as noted above, $KR > 1$, a very large number of multipole contributions have to be calculated numerically to ensure convergence of the final results. It has been shown [15] that it is sufficient to calculate only the inner-shell contributions by S-matrix methods and to evaluate the outer-shell contributions by simpler approaches. When $h\nu$ is close to an inner-shell threshold B, "spurious" resonance contributions to a given inner-shell amplitude have to be subtracted and an energy shifting procedure indicated by Eq. (18) has to be adopted [2].

$$(h\nu - B)_{\text{expt}} = (h\nu - B)_{\text{theory}} . \tag{18}$$

Note that all the above mentioned calculations of Rayleigh scattering have been performed in the independent particle approximation (IPA).

C. Delbrück Scattering

In the lowest non-vanishing order, a typical graph for Delbrück scattering involves two virtual photons in the Coulomb field of a nucleus. The resulting amplitude of order $(Z\alpha)^2\alpha$ has been calculated by Paptzacos and Mork [16], where α is the fine structure constant (~1/137). Furry's theorem tells us that an odd number of electron lines in closed loops do not contribute to the final scattering amplitude. Therefore, Coulomb corrections involve an even number of virtual photons and lead to terms of order $(Z\alpha)^4\alpha$, $(Z\alpha)^6\alpha$, etc. In the regime of high $h\nu$ and small θ, these have been calculated by Cheng and Wu by standard quantum-electrodynamic procedures [17], and in a semi-classical approximation by Milstein and Strakhovenko [18]. At energies of several GeV and angles of 1-3 milliradians, the corrections reduce the calculated cross sections by factors of the order of 5. Note that the Coulomb corrections have not yet been calculated at much lower energies at which several experimental studies have been performed.

D. Nuclear scattering

Due to the complexity of calculations of the nuclear scattering amplitude and the limitations of space, these calculations [19-22] including considerations of giant resonances, center of mass motion, exchange currents, etc. are not discussed here further.

4. Discussion of Experimental Results

A. Nuclear Thomson scattering

The elastic scattering of 1.6 MeV γ rays of a (^{140}Ba + ^{140}La) source through 124° by hydrogen, lithium, carbon and aluminium was studied by Alvarez et al. [23]. Rayleigh and Delbrück contributions are very small in this study. The Z^4/A^2 dependence of the small nuclear-Thomson-scattering cross sections ($< 10^{-30}$ cm^2/sr) was confirmed to about $\pm$10%.

B. Rayleigh scattering

A very large number of experiments performed with γ rays of energies up to about 4 MeV have provided considerable information concerning Rayleigh scattering. The theoretical calculations have been confirmed fairly well, although the cross section varied from about 10^{-22} cm^2/sr at 1° in the case of high-Z targets to about 10^{-27} cm^2/sr at large angles in the case of low-Z targets.

Firstly we will consider the regime of small x and medium or high Z, in which Rayleigh scattering is the dominant contribution. The cross sections for the scattering of 1.33-MeV γ rays through 4.5° to 12° by tin are shown in Fig. 3. The cross sections calculated on the basis of RFF and indicated by the solid line are significantly larger than the experimental values [24]. The values calculated on the basis of the MF approximation are indistinguishable on the scale of the figure from realistic S-matrix calculations and are in agreement with the experimental data. Similar data obtained in the case of a lead target by two groups [24,25] are presented in Fig. 4. Here, the ratio of the experimental cross section to the Rayleigh cross section calculated in the MF approximation is shown as a function of x. The inclusion of nuclear Thomson (~ +1%) and Delbrück scattering (~ -3%) amplitudes should lower the calculated ratio below unity, as indicated by the region of slanting lines. The errors are not shown in the case of data of Ramanathan et al. in order to ensure clarity of the figure. The experiments are seen to be in reasonable agreement with theoretical calculations. Of course, a reduction of the experimental error is necessary before the Delbrück contribution can be established in this regime. Scattering of γ rays of 0.344-1.408 MeV through small angles [26] and of 1-4 MeV through 1° and 1.8° [27] also indicate similar agreement for $x > 1$ Å^{-1}. Some of the deviations seen at lower values of x are probably due to the difficulty of separation of unresolved Compton and elastic scattering signals and to the possible structure dependent effects even in polycrystalline targets.

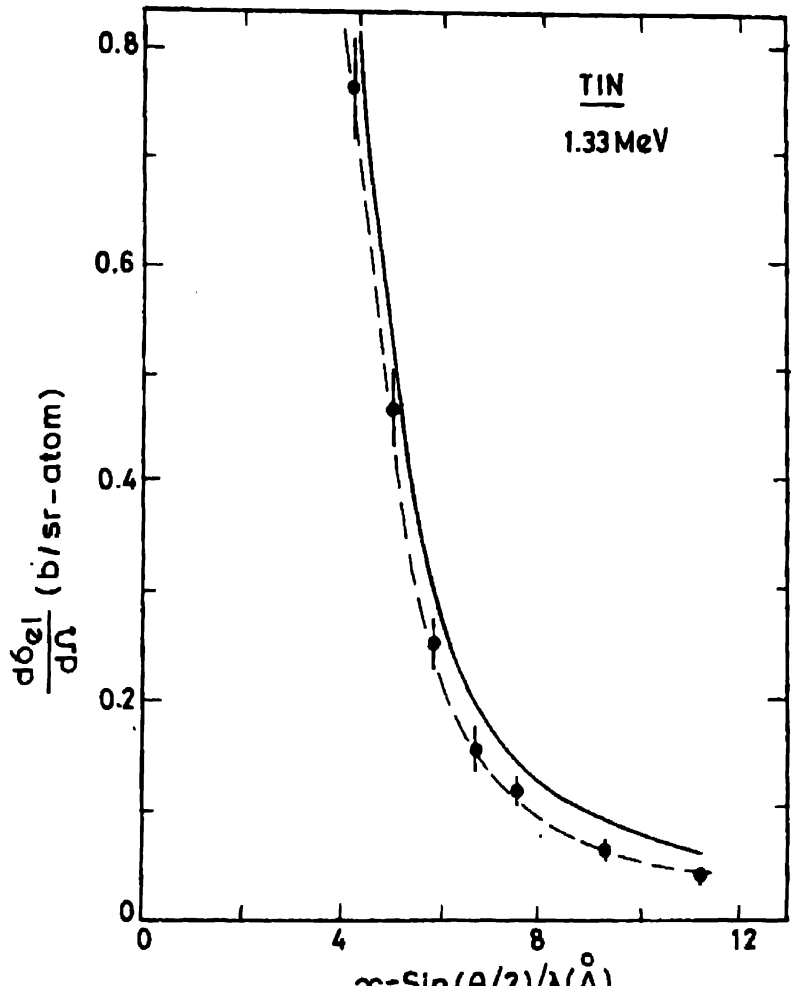

Fig. 3. The differential cross section for elastic scattering of 1.33 MeV γ rays by tin through small angles as a function of x [24]. The MF approximation, but not the RFF, is seen to be satisfactory in this regime.

Next, we will examine recent studies at photon energies close to K-shell thresholds. A ^{109}Cd source gives γ rays of 88.03 keV, only 27±7 eV above the lead K-shell binding energy. The experimental values of cross sections [28] at an angle of 125° in the case of gold, lead and bismuth are 0.71±0.05, 0.60±0.055 and 0.17±0.017 b/sr, respectively, and are in fair agreement with

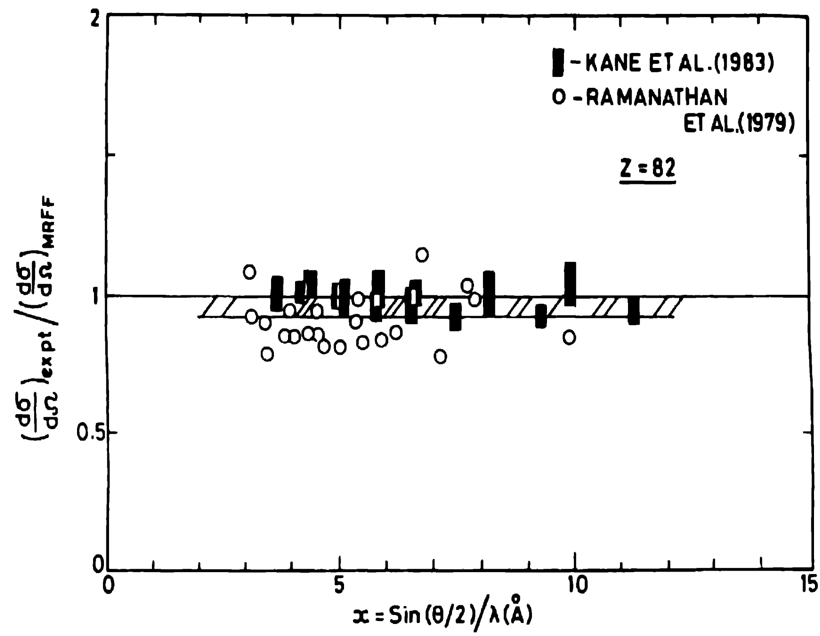

Fig. 4. Experimental cross sections for elastic scattering of γ rays of different energies by lead through small angles [24, 25] are compared with calculations of Rayleigh scattering based on the MF approximation. The region of slanting lines indicates the values expected on the basis of a combination of Rayleigh, nuclear Thomson and lowest order Delbruck contributions.

the S-matrix calculations of the Pittsburgh group. Elastic scattering of 81.0 keV γ rays of a ^{133}Ba source by five elements was studied in the angular range from 45° to 145° [2]. The K-shell binding energy of gold is only about 275 eV below this photon energy. As shown in Fig. 5, the separation of counts arising from $K\,\beta_2'$ x-rays of gold and from 81 keV elastic scattering has to be done by a least-squares-fitting procedure. The angular distributions of cross sections are shown in the case of aluminium and lead in Fig. 6, and in the case of tantalum and gold in Fig. 7. As expected from the discussion in Sec. 3, the MF approximation is seen to be unsatisfactory in the case of a high-Z target near the K-shell threshold. The (MF+ASF) approximation with angle independent anomalous scattering factors is better but predicts cross sections which are slightly larger than experimental values at large angles. The S-matrix calculations are found to be in fair agreement with the experimental data throughout the angular range investigated.

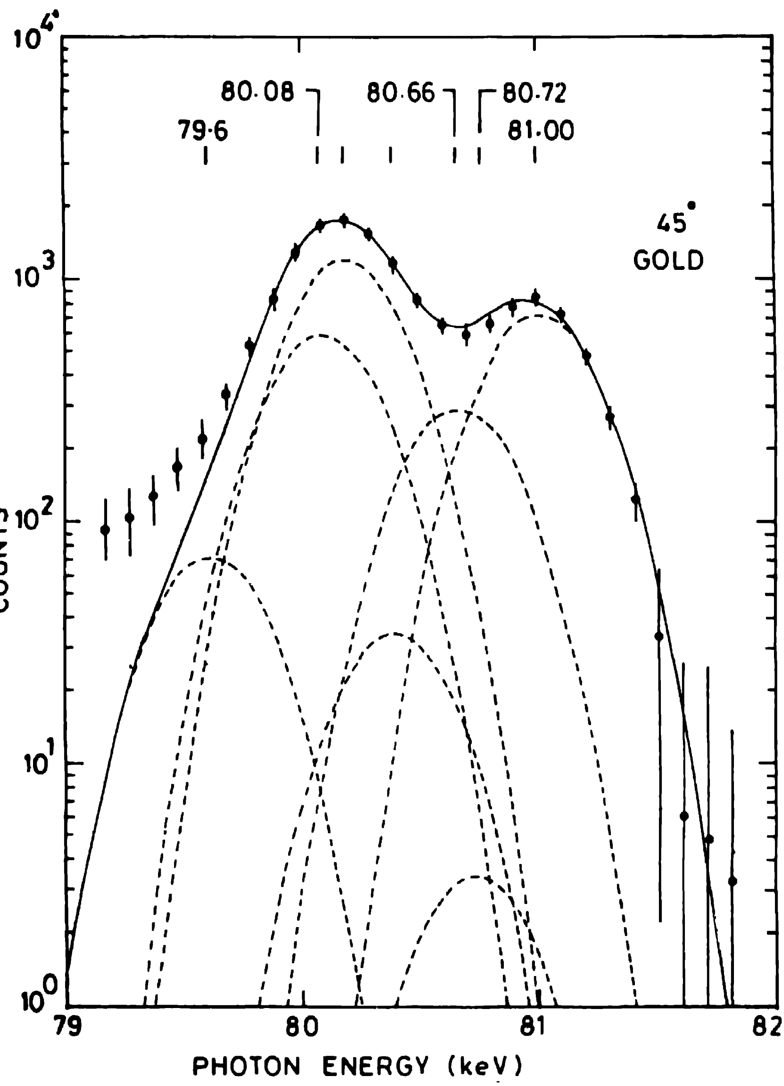

Fig. 5. HpGe detector spectrum near 81 keV after scattering of ^{133}Ba γ rays through 45° by gold with the K-shell binding energy of 80.725 keV [2]. The dashed lines indicate seven contributions arising from main $K\ \beta_2{}'$ x-rays of gold, and elastically scattered γ rays of 79.6 and 81.0 keV. The separate contributions were determined by a least-squares-fit of the experimental data, the fit being indicated by the solid line. The fit is poor at the lowest energies shown because of the tail of the more intense $K\ \beta_1{}'$ x-rays not included in the fitting procedure.

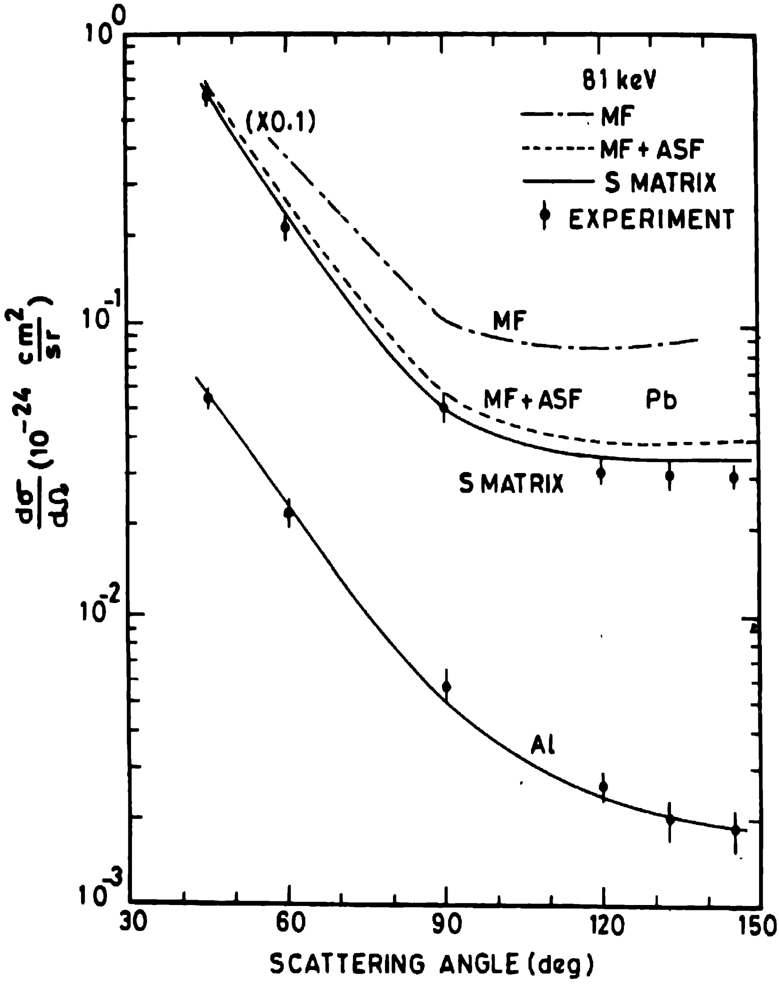

Fig. 6. A comparison of experimental cross sections for elastic scattering of 81.0 keV γ rays by lead and aluminum with cross sections calculated by the addition of nuclear Thomson and Rayleigh amplitudes. Calculations based on the MF, the (MF + ASF) and the S matrix approaches are indistinguishable on the scale of the figure in the case of aluminium.

C. Delbrück scattering

In the case of large-angle scattering of γ rays of about 1 MeV by a high-Z target, the nuclear Thomson amplitude (~ 5×10^{-15} cm) is about half of the

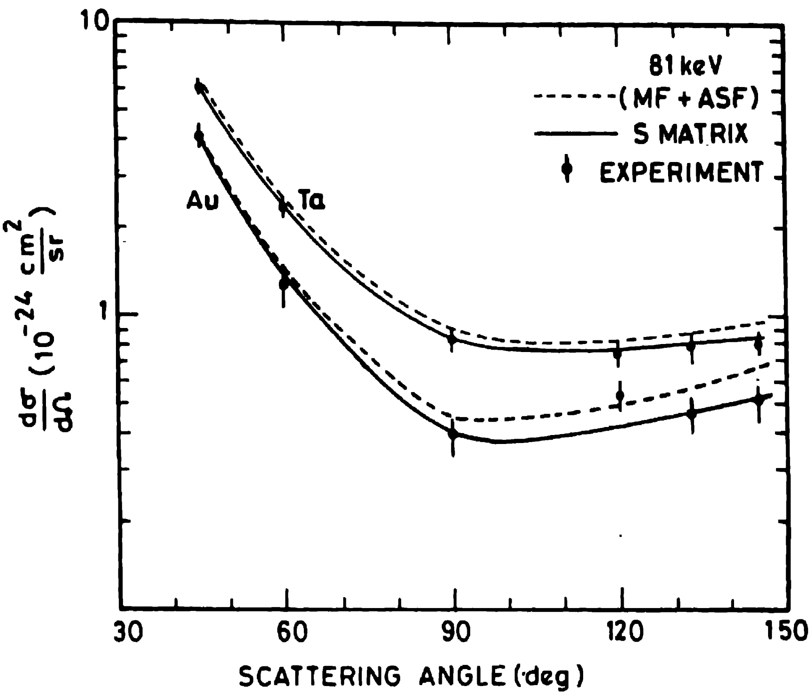

Fig. 7. Similar to Fig. 6 but for tantalum and gold.

Rayleigh amplitude, the real Delbrück amplitude is about 10% of the Rayleigh amplitude and the imaginary Delbrück amplitude is very small. The Ge(Li) detector results of three groups for the scattering of 1.33-MeV γ rays by a lead target [29,30,31] are shown together in Fig. 8. The different experimental data are seen to be in agreement with each other and with calculations of Rayleigh scattering based on the S-matrix approach and of Delbrück scattering in the lowest non-vanishing order. Thus a combination of experimental data and theoretical calculations of different groups led to a clear evidence for real Delbrück amplitudes.

An earlier experiment [1] concerning elastic scattering of 1-7 GeV photons through 1-3 milliradians (< 0.18°) by copper, silver, gold and uranium provided a clear evidence for the imaginary Delbrück amplitudes and also for substantial Coulomb corrections. See Sec. 3C for details. The Rayleigh cross section is very small in comparison to the Delbrück cross section of about 125 b/sr for 5-GeV photon scattering by gold through 1 mrad. However, a correction of the order of 10% had to be made for the contribution of Compton scattering to the measured counts, since the experimental resolution of about

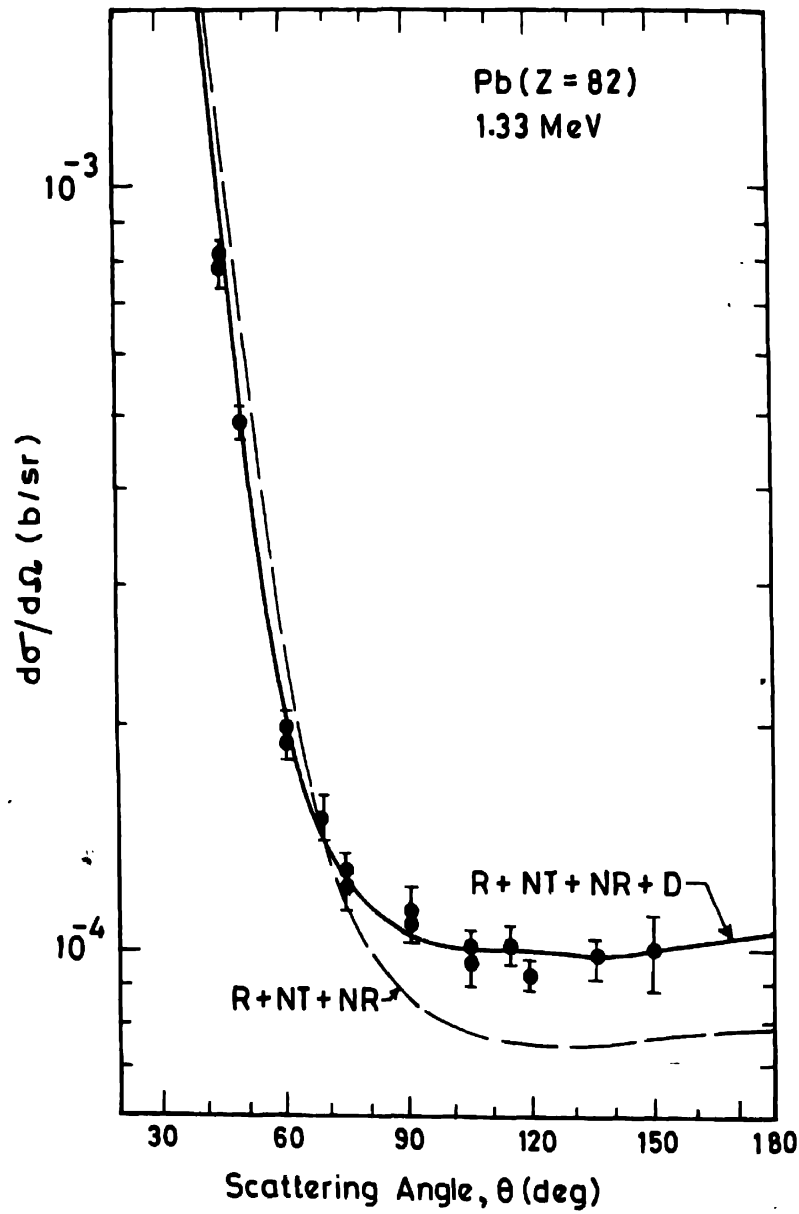

Fig. 8. Experimental values of cross sections for elastic scattering of 1.33 MeV γ rays by lead [29, 30, 31]. The calculations of the Pittsburgh group are indicated as (R+NT+NR) and (R+NR+NR+D); i.e. without and with the inclusion of Delbrück scattering.

$\pm$1.5% amounted to about 75 MeV at 5 GeV.

Similar conclusions were reached through experimental studies of elastic scattering of about 9-MeV photons by tantalum, lead, bismuth and uranium [32,33,34]. Near 40°, the Rayleigh, the nuclear and the real Delbrück amplitudes are small and the scattering cross section is dominated by the imaginary Delbrück contribution. The tantalum data (Z=73) confirmed the lowest-order [16] Delbrück contributions. The cross sections determined in the case of uranium (Z=92) suggested an angle-dependent decrease by up to a factor of 2 in comparison with the lowest-order Delbrück calculations. Note that Coulomb corrections have not been calculated in the energy regime under consideration or at lower energies.

A series of experiments have been performed by the Göttingen group concerning elastic scattering of photons of 2.754 MeV and of higher energies up to about 4 MeV [35]. In this energy range, a large number of amplitudes have to be taken into account in the calculations of cross sections. Experimental data with errors of about $\pm$10% were compared with calculations of Rayleigh, nuclear and lowest order Delbrück components and Coulomb corrections of order $(Z\alpha)^4\alpha$ were deduced empirically with the assumption of relations (19) and (20).

$$\text{For any } \theta, \; \frac{B_{\mathrm{D}\parallel}}{B_{\mathrm{D}\perp}} = \frac{A_{\mathrm{D}\parallel}}{A_{\mathrm{D}\perp}}, \tag{19}$$

$$\text{and for } \theta = 180^\circ, \; \mathrm{Re}\, B_{\mathrm{D}\parallel} = \mathrm{Im}\, B_{\mathrm{D}\parallel} \text{ and } \mathrm{Re}\, B_{\mathrm{D}\perp} = \mathrm{Im}\, B_{\mathrm{D}\perp}, \tag{20}$$

where A and B indicate the lowest non-vanishing order Delbrück amplitudes and Coulomb corrections, respectively. The relative effect of Coulomb corrections on the cross sections was found to be largest near 60°, additive and of the order of 20% in the case of uranium.

D. Nuclear scattering

The first excited states of ^{12}C, ^{16}O and ^{208}Pb nuclei are at 4.4, 6.06 and 2.6 Mev, respectively, and so elastic scattering is separated relatively easily from inelastic scattering in these cases. Therefore, scattering from these nuclei has been studied more extensively. The roles of giant dipole and electric quadrupole resonances have been investigated in several experiments [36,37,38,39]. The effect of nuclear form factor has been identified in the study of scattering of 15-100 MeV photons through 90° [40], and of 3-6 GeV photons through 10-50 milliradians [41]. A detailed discussion of such results is not presented here. It can be seen in references [19] to [22].

5. Further Studies

Additional studies in the 70-100-keV range will be very useful for a further confirmation of the success of S-matrix calculations of Rayleigh scattering, based on the independent particle approximation, even very close to K-shell thresholds of high-Z atoms. Theoretical calculations of Coulomb corrections to Delbrück scattering amplitudes at energies lower than 100 MeV are desirable for developing greater confidence in the magnitudes of the empirically deduced Coulomb corrections. The experimental efforts of the Göttingen group in the 2-4 MeV range should be supplemented by measurements in other laboratories. If the different factors mentioned in Sec. 2 in connection with the determination of cross sections are measured with greater accuracy than heretofore, it should become possible to determine the cross sections to about $\pm 1\%$ and thereby to improve our understanding of the elastic scattering processes.

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